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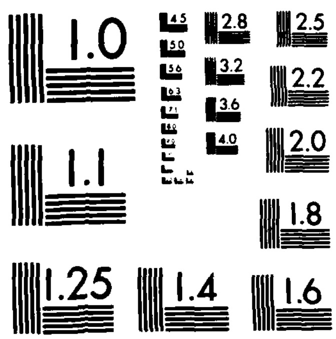

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FINAL REPORT ON THEORY OF SOLITON WAVES

HARVEY SEGUR

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Princeton, New Jersey 08540

November, 1984

Prepared for

Office of Naval Research
800 N. Quincy Street
Arlington, VA 22217

Contract N00014-80-C-0048, NR 062-585

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The long-range objective of the work performed under this contract has been to construct realistic models of water waves using solitons and other intrinsically nonlinear waves as the fundamental building blocks. In principle, both deterministic and statistical models could be constructed in this way, in deep water or in shallow water. The basic idea is that because water waves are known to be fundamentally nonlinear, one has a better chance of describing them with mathematical models that are also fundamentally nonlinear. This report describes the progress we have made in achieving that objective. and keywords include:

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A. INTRODUCTION

The long-range objective of the work performed under this contract has been to construct realistic models of water waves using solitons and other intrinsically nonlinear waves as the fundamental building blocks. In principle, both deterministic and statistical models could be constructed in this way, and the models might be appropriate either for surface or for internal waves, in deep water or in shallow water. The basic idea is that because water waves are known to be fundamentally nonlinear, one has a better chance of describing them with mathematical models that are also fundamentally *nonlinear*. *This report describes the progress* we have made in achieving that objective.

B. THE BOTTOM LINE

Here is a very brief report on our progress. The current contract has been funded since 1980. During that time, the Principal Investigator has published 12 papers and one book dealing with aspects of nonlinear models of water waves. These are listed in §G of this report. The most important paper on this list is that of Segur & Finkel (1984). It gives an analytical model of periodic waves in shallow water. This model has 8 free parameters, and an explicit algorithm is given to deduce these 8 parameters from 8 specific measurements of the wave. In this sense, the model is completely deterministic. The waves described by this model are intrinsically nonlinear, and they are periodic in two independent horizontal directions. Just as cnoidal waves are often used as one-dimensional models of "typical" periodic

waves in shallow water, this model can be used to describe "typical" periodic waves in shallow water when the restriction of one-dimensionality is relaxed,

To our knowledge, this is the first practical model ever of periodic waves in shallow water, in which the waves are intrinsically nonlinear and two-dimensional. The point is not that the model is more accurate than its competitors, but rather that it has no competitors. In this respect, the research program effected under this contract has been completely successful. It is unfortunate that the program has been terminated just when its practical benefits were about to be realized.

C. BACKGROUND

A more complete accounting of the work done under this contract requires a broader perspective, because the subject of water waves is not restricted to surface waves in shallow water. We begin the more comprehensive review by defining a basic concept (integrability) that will be used in the remainder of this report. A partial differential equation is said to be integrable if it can be solved exactly as an initial value problem, with arbitrary initial data in a certain class. Most linear equations are integrable in this sense, but integrable nonlinear equations are uncommon. However, it happens that several nonlinear equations that are known to be integrable are also models of water waves in various contexts. Among the one-dimensional models of water waves (i.e., models in which the wave pattern varies in one spatial dimension and in time), the following equations all are integrable.

1. The Korteweg-deVries (KdV) equation,

$$u_t + 6uu_x + u_{xxx} = 0 \tag{1}$$

describes the evolution of long surface waves of moderate amplitude as they propagate without dissipation in only one direction in relatively shallow water (Korteweg & deVries, 1895). It also describes the evolution of long internal waves under similar circumstances.

2. For internal waves, there are several meanings of "long". The KdV equation applies if the horizontal wavelength is long in comparison with the total fluid depth (Benney, 1966). If it is long in comparison with the thickness of the appropriate pycnocline, but small in comparison with the total depth, then the appropriate evolution equations is due to Benjamin (1967) and Ono (1975),

$$u_t + uu_x + H\left(u_{xx}\right) = 0, \tag{2}$$

where H (•) denotes the Hilbert transform. An intermediate equation that interpolates between these two is due to Joseph (1977) and Kubota, Ko and Dobbs (1978),

$$u_t + uu_x + T\left(u_{xx}\right) = 0, \tag{3a}$$

where

$$T(f) = \frac{1}{2} \fint_{-\infty}^{\infty} \coth \frac{\pi}{2} (y - x) f(y) \, dy, \tag{3b}$$

and the integral is evaluated in the principal-value sense.

3. Long internal waves also admit a degenerate configuration in which the (scaled) coefficient of the nonlinear term in (1) exactly vanishes. In this case, the KdV equation should be replaced by the so-called modified Korteweg-deVries equation (mKdV),

$$u_t - u^2 u_x + u_{xxx} = 0, \tag{4}$$

as shown by Djordjevic & Redekopp (1978).

4. Marginally unstable baroclinic waves on a beta-plane satisfy the sine-Gordon equation,

$$\phi_{xt} = \sin \phi , \tag{5}$$

as shown by Gibbon, James & Moroz (1979).

5. The fundamental nonlinear interaction between internal and/or inertial waves of moderate amplitude is called a resonant triad, or three-wave interaction (Phillips, 1976). The equation describing the interaction of exactly three resonant wave packets is

$$\partial_t A_1 + c_1 \partial_x A_1 = i \gamma_1 A_2^* A_3^*, \tag{6}$$

$$\partial_t A_2 + c_2 \partial_x A_2 = i \gamma_2 A_3^* A_1^*,$$

$$\partial_t A_3 + c_3 \partial_x A_3 = i \gamma_3 A_1^* A_2^*,$$

where A_j is the complex amplitude of the envelope of the j^{th} packet, with linear group velocity c_j, and $\gamma_1 \gamma_2 \gamma_3 < 0$.

6. The nonlinear Schrödinger equation,

$$i A_t + A_{xx} + 2 \sigma |A|^2 A = 0, \qquad \sigma = \pm 1 , \tag{7}$$

describes the evolution of a packet of a nearly monochomatic surface waves in sufficiently deep water (Zakharov, 1968). This equation also describes the evolution of a nearly monochromatic packet of edge waves (Whitham, 1976).

All of these equations were derived from the known inviscid equations of surface or internal water waves by taking various limits. That each of the models should be integrable is quite remarkable. A consequence of this integrability is that each of these equations admits various families of exact solutions, which we now discuss. <u>Solitons</u> are spatially localized, traveling waves that retain their identity despite interaction with other localized waves. For the KdV equation, a single soliton is simply the solitary wave that was found by Korteweg-deVries:

$$u(x,t) = 2 \kappa^2 \operatorname{sech}^2 \left\{ \kappa (x - 4 \kappa^2 t + x) \right\} . \tag{8}$$

The integrability of (1) guarantees that it admits not only (8) as a solution, but also more complicated exact solutions, called N-soliton solutions (Gardner, Greene, Kruskal, Miura, 1974). These can be viewed as exact, nonlinear superpositions of N solitary waves. It is because solitons can be combined exactly in this way that we can hope to use them as the fundamental building blocks in a new, intrinsically nonlinear model of water waves.

All of the integrable equations listed above admit solitons of some sort, and this same argument applies to any of the soliton solutions. In every case, the solitons are localized in space.

Korteweg & deVries also discovered a periodic traveling wave solution of (1), which they called a <u>cnoidal wave</u>:

$$u\,(x,t) = 2\,p^2\,k^2\,\mathrm{cn}^2\left[\,p\left(x - ct + x_o\right)\,;\,k\right] + u_o, \tag{9}$$

where $\mathrm{cn}\,[\Phi;k]$ is Jacobian elliptic function with modulus k. Like the solitons, these periodic waves can be embedded in families of more complicated solutions, called <u>N-gap solutions</u>. These solutions are quasi-periodic functions of space and time, and they can be written in the form

$$u\,(x,t) = 2\,\partial_x^2\,\ln\,\theta, \tag{10}$$

where θ is a Riemann theta function of genus N. The N-soliton formulae follow from these quasi-periodic solutions by taking an appropriate (infinite-period) limit.

The KdV admits a third kind of special solution,

$$u\,(x,t) = (3t)^{-2/3}\,f\left(x/(3t)^{1/3}\right), \tag{11}$$

where $f(\eta)$ satisfies a nonlinear ordinary differential equation, and is related to one of Painlevé's transcendents. The important role of these self-similar solutions in the long-time ($t \to \infty$) asymptotic solution of (1) was shown by Ablowitz & Segur (1977), and verified experimentally in water wave

experiments by Hammack & Segur (1978).

Thus, for the KdV equation we have exhibited three different kinds of exact solutions, any or all of which might serve as building blocks in a new model of water waves. As one might expect, each of the integrable equations listed above has its own set of special solutions, which are discussed in detail by Ablowitz & Segur (1981). It remains to use these special solutions self-consistently to construct realistic models of water waves.

D. THE MASTER PLAN

What steps are required to construct realistic models of water waves, based on these special solutions of integrable equations? A logical and prudent sequence of steps is the following:

1. Identify an integrable evolution equation in (1 + 1) dimensions that describes approximately the evolution of surface, internal and/or inertial waves under appropriate conditions. (One can regard equations (1)-(7) as the cumulative result of work on this step.)

2. Solve the initial value problem for arbitrary smooth initial data that:

   a) vanish rapidly enough as $x \to \pm \infty$; or
   b) are periodic on a finite interval.

These are two different problems and they require different methods of solution. Here "solve" means to prescribe an explicit algorithm to construct the solution of the evolution equation at any finite time that evolved from the given initial data. To be effective, the algorithm must be uniformly valid in time, and this requirement excludes the possibility of direct numerical integration of the evolution equation.

3. The general solution of the initial value problem is generally too complicated to be of practical value. However, the general solution can be approximated by combinations of the special solutions listed above (solitons,

similarity, and quasi-periodic solutions) in a way that is consistent with the dynamics.

a) For problems posed on $-\infty < x < \infty$, the solution usually simplifies in the long time $(t \to \infty)$ limit. Determine the long-time behavior of the general solution.

b) For problems with periodic boundary conditions on finite interval, the solutions are recurrent, so there is no long-time limit. For these problems, prescribe an effective method to extract important dynamical information from the general solution. For example, given some initial data for one of the evolution equations listed above, determine the first recurrence time.

4. All of the integrable evolution equations listed above are non-dissipative, whereas real water waves are slightly viscous. Determine the effect of weak viscous damping on these special solutions.

5. At this point, one can make precise predictions about the evolution of real water waves, provided that the waves are constrained to evolve in only one spatial direction (by a narrow wave tank, for example). Determine the accuracy of this one-dimensional model in controlled laboratory experiments.

6. Most problems in water waves require models that are either two-dimensional (surface gravity waves, internal waves on a pycnocline) or three dimensional (general internal and/or inertial waves). Embed he integrable equation in (1 + 1) dimensions into a physically appropriate model of water waves in (2 + 1) or in (3 + 1) dimensions. This higher dimensional model may or may not be integrable.

7. Determine whether the one-dimensional solitons and other special solutions or solitons are stable to transverse perturbations. Unstable solutions probably cannot be used in a practical model of water waves.

8. Determine whether the higher dimensional model is integrable. If so, repeat steps 2-5 for the higher dimensional problem.

9. Construct practical deterministic models of waterwaves, using the stable, intrinsically nonlinear waves found by this approach.

10. Construct practical statistical models of water waves, using the stable, intrinsically nonlinear waves found by this approach.

E. CONCRETE RESULTS

How do the papers published under this contract fit into this master plan? These papers can be organized according to the types of waves being studied.

1. Surface waves in shallow water

In (1 + 1) dimension, the governing equation is (1), and steps 1-5 had been completed before the present contract began (see Hammack & Segur, 1978). A generalization of (1) to (2 + 1) dimensions is the equation of Kadomtsev and Petviashvili (1970):

$$\left(u_t + 6uu_x + u_{xxx}\right)_x = 3\,\sigma\, u_{yy}, \quad \sigma = \pm 1. \tag{12}$$

This equation turns out to be integrable for $\sigma = \pm 1$. For capillary-type waves, $\sigma = +1$. Segur (1982b) resolved an aspect of the initial-value problem for (12) with $\sigma = +1$, on $-\infty < x,y < \infty$. For gravity-type waves, $\sigma = -1$. A preliminary model of periodic waves in shallow water, based on (11) with $\sigma = -1$, was given by Segur, Finkel and Philander (1983). The final version was published later by Segur & Finkel (1984). As discussed in §B, this is probably the most important piece of work completed under this contract. It is a concrete example of how one can use nonlinear waves as the basic building blocks in a model of water waves.

2. Surface waves in deep water

Here the governing equation in (1 + 1) dimensions is (7). Segur (1981) found a simple rule for the viscous decay of the envelope solitons of (7). These envelope solitons are unstable to transverse perturbations, however, so their practical value is not evident, except in narrow wave tanks in laboratories.

3. Long internal waves on a pycnocline

This problem may have important naval implications, because of the possibility of submarine detection. Consequently, 5 papers were published on aspects of this subject. Ablowitz & Segur (1980) derived equations analogous to (2) and (3), but for periodic waves. Ablowitz, Fokas, Satsuma & Segur (1982) generalized (2) and (3) to (2 + 1) dimensions, and determined the transverse stability of their solitary waves. Segur & Ablowitz (1981) found the long-time ($t \to \infty$) solution of (4) and (5). The KdV equation, (1), describes both long internal waves and long surface waves, but the viscous decay rates of their solitons differ, as shown by Hammack, Leone & Segur (1982). The most important paper in this series was by Segur & Hammack (1982), who compared the predictions of (1) and (3) with experimental data, and resolved an outstanding puzzle about the validity of these two models.

4. General internal waves

Segur (1980) cleared up a popular misconception about how surface and internal waves interact. Segur (1983) created an irreversible, statistical model of internal wave interactions, based on solutions of the (3 + 1) dimensional generalization of (6). This model is not yet practical because of certain mathematical ambiguities that have not yet been resolved. Even so, it may be the first genuinely new generalization of Boltzmann's equation in the last fifty years.

5. Reviews

Segur (1982a) and chapter 4 of Ablowitz & Segur (1981) review how all of these theories fit together to give a (still incomplete) picture of how water waves evolve.

F. SUMMARY

Clearly the plan outlined in §D is ambitious. It is not surprising that the plan has not yet been completed for all of the models listed in §C. Fortunately, the plan was carried to the point of producing one finished model, given by Segur & Finkel (1984). The accuracy of this model must still be determined experimentally. Even so, because of the simplicity and the intrinsic strength of the model, we expect it to become as widely used as an engineering model of periodic waves in shallow water as the cnoidal wave has been heretofore.

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